## Guide to Week 5 Readings

#### Montana (1988)

The main reading is the classic (1988 IJRR) paper by D.J. Montana on the kinematics of rolling contact. The paper contains a lot of formal definitions. It may help to refer to the Week5 slides 2-8 for terminology.

Fortunately, things become much simpler when the objects are flat, cylindrical or spherical. We are familiar with the spherical coordinate frames from our knowledge of latitude and longitude on Earth.

* For the assignment, it is useful to start with Definition 3 and Definition 4 and the derivation that culminates in eq. (14) and (15).
* If you do the operations in equation (6), where fu(**u**) indicates that we take the partial derivative of a coordinate frame *f=(u, v)* with respect to *u*, etc.,you can derive the results in equation (14), where (*u, v*) are given for the case of a sphere in equation (11).
* Then you can use eq (7-9) to assemble the [K], [T] and [M] matrices. Again, notation like **xu(u)** means take partial derivative of vector **x** with respect to vector **u**.
* The sample Sympy script MontanaKmat.py goes through these steps for the [K] matrix and, after simplification, we get a result that matches [K] in eq. (15) for the case of a sphere. It is strongly suggested to go through this script, executing a few lines at a time, and satisfy yourself that you see exactly what is happening.
* Then for the first part of the assignment, you are asked to extend the script to get the results for [T] and [M]. The end results should match those in eq. (15). At this point, you understand how Montana builds his coordinate frames and uses Gauss map definitions to get the [K], [T] and [M] matrices that define surfaces.

Then read Section 3, which shows how to map from relative velocities of two bodies to the corresponding motions, on the surfaces, of the contact coordinate frames. Equations (16-20) are key.

As noted in **Example 2** on page 23, the [K] and [T] matrices for a flat surface are zero, and [M] is the identity matrix. (If you like, you can derive this from your solution above, using   
fvec = Matrix([u,v,w].) Note that Montana’s results (eq 40-43) are assuming 𝜓 = 0, which would only be true at the start of a rolling trajectory.

#### Okamura 2000

Allison Okamura researched tactile sensing on a hemispherical fingertip with rolling and sliding. In the Appendix A of her PhD thesis, linked in the course reading list, she rederives the Montana equations for her application for rolling. You might find her explanation a bit easier to read than the original paper.

#### [Yuan et al. 2020](https://ieeexplore.ieee.org/abstract/document/9197146) and [Tincani et al. 2012](https://ieeexplore.ieee.org/abstract/document/6385939)

Yuan et al. has been added to the reading list because it is highly relevant. Tincani et al. is less relevant but worth a glance. These papers provide a couple of nice examples of hands designed to exploit rolling contact for high object mobility. Note, however, that this approach often results in grasps that are somewhat less resistant to high pull-out forces than one would get using very soft fingers. Other relevant applications include [CMU’s Ballbot](https://www.ri.cmu.edu/robot/ballbot/) which was an inspiration for [Disney’s BB-8](https://disney.fandom.com/wiki/BB-8).